

The Study About the Temperature Distributing of the Sphere Medium Surface

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Abstract: The medical examinations by the infrared ray are used to examine some illnesses, and they are not harmful to us. The body surface is considered as the plane in the examinations, but the parts of our bodies are the curved surfaces and the results are more exact when we consider the bodies as the curved surfaces. The sphere is the most simple curved surface so we studied the sphere medium interface, the heat radiation temperature distribution of the sphere medium based on the experiments and put forward the theory to explain the temperature distributing. We hope to give the supplement for the earlier research through our research.

Keyword: TTM, temperature distribution, thermal infrared imaging, thermal conduction, heat transfer.

□. Background

The heat exchange can be transferred in three ways: Conduction, Convection, and radiation.

The heat exchange occurred when temperature values of the two objects or the two parts of one object are different.

The thermal radiations generally exist in the universe. The Stefan-Boltzmann theorem tells that the hemisphere total radiant energy of the black body E_b is the direct ratio of the four-power Kelvin temperature T . See at the equation 1 and σ is the Stefan-Boltzmann constant. If we can measure the radiant energy at some point of the black body, we can know the temperature at the point according to the equation 1.

$$E_b = \sigma \cdot T^4 \quad (1)$$

Our body are not the black body, but when the temperature is under the usual conditions, which the temperature is between 20 centidegree and 45 centidegree, we can

consider them as the black body. Then the constant is not the Stefan-Boltzmann constant but less than the Stefan-Boltzmann constant.

Then it is necessary to introduce the definition about the radiation angular coefficient F_{1-2} . Suppose there are two tiny black body surfaces S_1 and S_2 , and the α_1 and the α_2 are the angles about their normal directions and the direction of the conjoint line between the S_1 and the S_2 , and the L is the distance of the S_1 and the S_2 . The F_{1-2} is

$$F_{1-2} = \frac{\cos\alpha_1 \cdot \cos\alpha_2}{\pi \cdot L^2} \cdot dS_2 \quad (2)$$

The heat conductions in the body follow the Fourier theorem, which tells that q is the heat flow density in the temperature field in the isotropy and uniform medium, and its direction is contrary on the direction of the vertical temperature gradient and the value is equal to the product of the coefficient of the thermal conductivity λ and the vertical temperature gradient. In the equation 3 T is the Kelvin temperature and it is the same that we use the Kelvin temperature or the other temperature t .

$$q = -\lambda \cdot \text{grad}(T) \quad (3)$$

The steady temperature field where there are no thermal sources can be explained through the laplace equation, and t is the temperature of the point whose coordinate is x, y, z in equation 4.

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} = 0 \quad (4)$$

If we have the boundary conditions, the equation can be solved. But it is usually impossible that the temperature analytic solution is found, so usually we can only get the numerical value unless the temperature field interface is very simple.

□. CURRENT RESEARCH STATUS

The thermal infrared imagery is the image on the surface temperature distributing. The surface temperature distributing can offer the information of the exothermic resource in the body and the transferring heat path from the thermal source to the interface of the body. Some illnesses can bring the abnormal metabolism that is just the heat resource, so the illnesses can be examined through the thermal infrared imageries.

In the 1970s, it is very popular that the thermal infrared imagery were used for the diagnosis. Many people studied the temperature distribution of the bodies in order to get the potential law of the temperature distribution for application in medical diagnosis. But this law is very complicated. However, a few people continued to study it, because they knew the value of infrared in medical examinations. It is non-invasive and non-ionizing, and hence not harmful to humans. In the later 1990s, the researchers in the Bioyear Company gave a simple way to get the inner temperature information according to the surface temperature distributing and now the way has been applied to explain the surface temperature distribution.

□. PROBLEM:

The way the researchers in the Bioyear Company have given is based on the hypothesis that the surface is plane. But the body surface is not plane and more

approximate to the sphere. Nobody has yet given the temperature distributing about the sphere medium. This article is the research about the temperature distributing of the sphere medium surface. We hope to use our research in the Bioyear way to get the more accurate result.

□.METHODS

At first the process should be introduced: the thermal source in the sphere medium sends out the heat energy, and the energy brings the temperature distributing of the interface when it gets to the interface by the thermal conduction. The thermal radiation from the interface go into the space and the infrared detector will scan the part of the heat radiation. So the process is divided into two parts: one is the process of the conduction and the other is the process of the radiation. We can ignore the atmosphere thermal radiation because it is very thin and the distance from the sphere to the detector. When we know the processes clearly, we shall get the proper model of the sphere temperature-distributing image by the equation 1.

1. The thermal radiation from the interface to the detector

Because there are many factors that affect the result, detecting the thermal radiation is very complex and it cannot be solved if all factors are thought over, so we must give some approximate ways to predigest the problem.

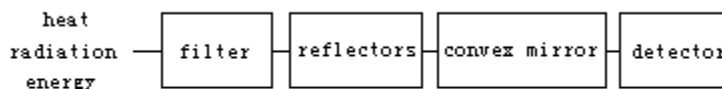


Figure 1.

At first the infrared energy should get through the optical system that is made up of a filter, some reflectors and a lens before it gets to the sensor. The filter is the infrared filter getting rid of the other light except the infrared light, and the reflectors and the convex mirror can lead the energy assembling into the infrared detector. These reflectors are necessary because the detector is fixed and they can assemble the light from the different dot of the space into the detector when the reflectors turn according to the period.

So we suppose a turned detector to replace

the optical system made up of the reflectors and lens in order to predigest the problem. The detector can turn flatly to scan the 256 points and then turn vertically to the next line beginning to scan the new 256 points. When it finishes scanning 256*256 points, it will return to the beginning place. In the picture 2 R is the radius of the sphere medium; the L is the distance from the detector to the sphere center and the L' is the distance from the measured point to the detector. The horizontal turning direction is not drawn in the picture 2.

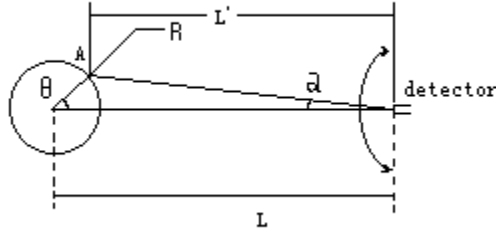


Figure 2.

In fact, the radius only is a few centimeters length, but the distance from the sphere center to the detector is about tow meters. So we suppose the R is 5 centimeters length and the distance L is 2 meters length. When it can scan the sphere, the detector will turn the maximal angles that is

$$\alpha_{\max} = \arctg \frac{0.05}{2} = 1.43^\circ$$

When the $L \gg R$, the equation is changed as follows

$$dQ_A = (E_A - E_{\text{sensor}}) \frac{\cos(\theta + \alpha) \cos\alpha}{\pi(L)^2} dS_{\text{sensor}} dS_A$$

We can prove the next equation:

$$\cos(\theta + \alpha) \cos\alpha \approx \cos\theta$$

So we can use a group of detectors to replace the turning detector. In the picture 3 $L \gg R$ and the radiation from the point A_1 is received by the detector lying in the point A_2 . The sizes of all detectors are uniform and w width and h length.

(5) Suppose the E_A is the hemisphere total radiant energy of the point A, the E_{sensor} is the hemisphere total radiant energy of the detector, the $F_{A-\text{sensor}}$ is the radiation angular coefficient. Then the energy the detector receives is

$$dQ_A = (E_A - E_{\text{sensor}}) F_{A-\text{sensor}} dS_A$$

The sphere center is the point O that is supposed as the origin of rectangular Cartesian coordinates. So every of them will only receive the horizontal infrared.

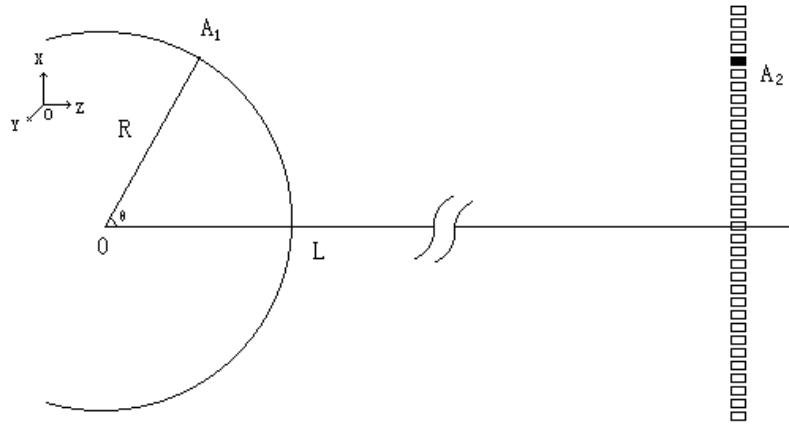


Figure 3.

If the E_1 is the radiant energy at the point A_1 and the E_2 is the radiant energy at the point A_2 , we can get equation 5 because $L \gg R$ and we can think the distance from the point A_1 to the point A_2 as the length L.

$$Q = \iint (E_1 - E_2) \cdot F_{1-2} \cdot dS_{A1} \quad (6)$$

This equation describes that the value of the energy the detector receives is a curving

surface integral about the radiant difference and the radiation angular coefficient F_{1-2} and the differential of the curving surface at the point A_1 . The S_{A1} is the projection of the detector A_2 . So we can get the equation as follows

$$Q = (E_1 - E_2) \cdot \frac{w \cdot h}{\pi L^2} \iint \cos\theta \cdot dS_{A1}$$

$$\begin{aligned}
&= (E_1 - E_2) \cdot \frac{w \cdot h}{\pi L^2} \int_x^{x+w} \int_y^{y+h} \cos \theta \cdot \sqrt{1 + \left(\frac{dz}{dx}\right)^2 + \left(\frac{dz}{dy}\right)^2} dx dy \\
&= (E_1 - E_2) \cdot \frac{w \cdot h}{\pi L^2} \int_x^{x+w} \int_y^{y+h} 1 dx dy \\
&= (E_1 - E_2) \cdot \frac{w^2 \cdot h^2}{\pi L^2} \\
&= K(E_1 - E_2)
\end{aligned}$$

the result is the equation 7

$$Q = K(E_1 - E_2) \quad (7).$$

The equation 7 tells us that the infrared energy the detector receives is only related with the radiant difference of the object and the detector. The detector can measure the difference

and usually the E_2 is not changed at one examination, so the E_2 is the const.

So the radiant energy is same when the temperature of the surface is same, and the conclusion is accordant with that of the plane. The measured result is related with the temperature at the same point on the sphere and it is not affected on the location on the sphere.

2. the law of the thermal conduction in the medium:

The law of the steady heat conduction in the body follows the laplace equation when there is no heat resource in the medium. When there is the heat resource the equation is

$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} = -\frac{q_v}{\lambda}$. The q_v is the heat energy produced by one volume medium.

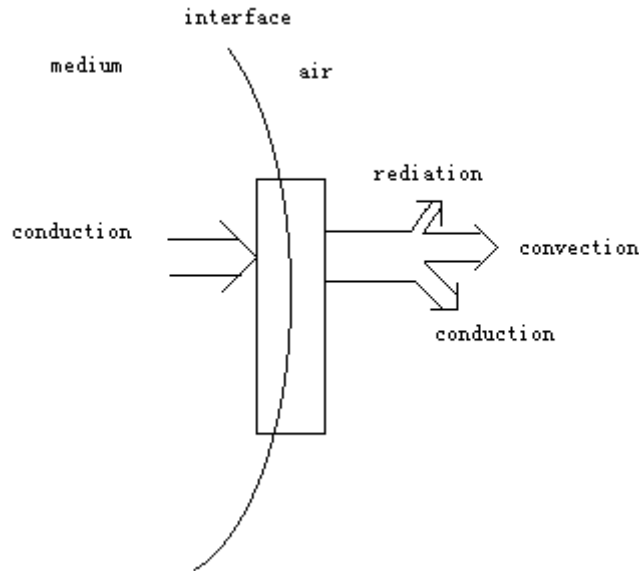


Figure 4.

The conditions of the interface are too complex. The picture 4 tells the conduction energy from the medium is divided into three parts that are the air thermal radiation whose part energy will receive by the detector, the air thermal convection that is the main way scattering the heat and the air thermal conduction that is weaker than that of the other ways. It is according to the law of conservation of energy.

The equations of the conduction and the radiation is forenamed (seeing the equation 1 and 2) and that of the convection is as follows:

$$q = \alpha(T - T_p)$$

The q is the heat flow density, the T is the Kelvin temperature of the medium surface, the T_p is the Kelvin temperature of the air and the α is the const.

We have got the equation and the boundary conditions but the analytic answer is difficult to get. So we shall use the other way to solve the problem.

At first we must understand fully the heat flow process. The heat from the heat resource conducts to the interface, and then goes into the air. There is a transient process exchanging the heat in the air, and the air near the interface is heated and its temperature is increased. So we think the process in the air is isotropic to predigest the process and the heat easily dissipate in the air, which is very important. If we use the hypothesis we can apply the new simple way to get the heat conduction result.

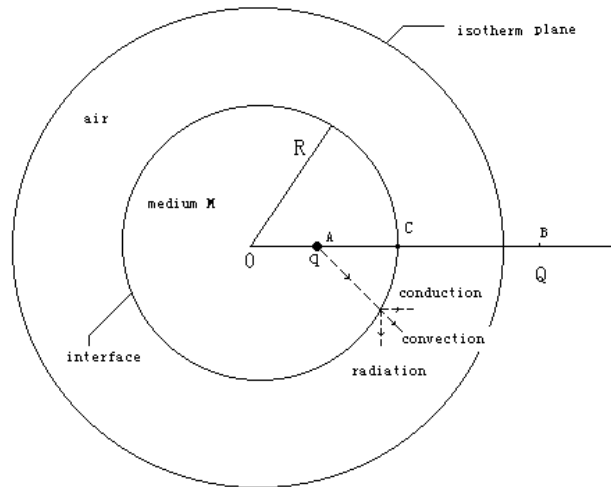


Figure 5(a).

The process is as the picture 5a. The R is the sphere radius OC in the picture, and we use the electrostatic field to simulate the process. The temperature is replace by the electric potential, and the energy stream density is

instead of the electric field intensity. The air easily absorbs the heat. So the thickness from the interface to the isotherm plane is very isotropic, and the model is like the picture 5 b.

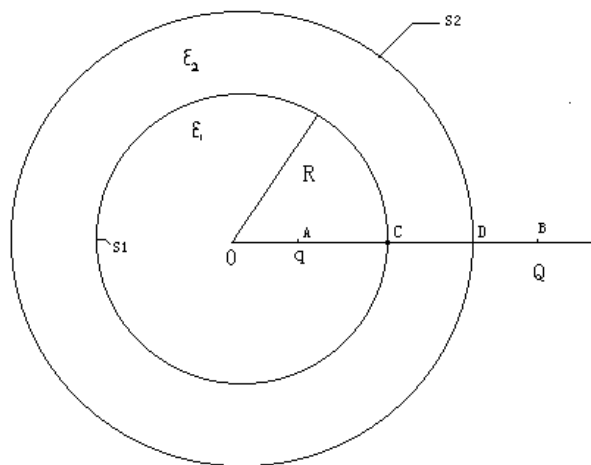


Figure 5(b).

We suppose the air layer is so isotropic that we can ignore the asymmetry emitting the heat, then we shall get tow concentric sphere S₁ and S₂. Let the constant of the S₁ and the S₂ are the ε₁ and the ε₂. The ε₁ is much bigger than the ε₂ and the layer from the S₁ to the S₂ is ignored. The S₂ is equipotential. The model is very simple and familiar and we can get the distributing of the electric field intensity.

We use the electrostatic field to do it, and place an electric charge Q to the point B in order to keep the electric field intensity not changing in the sphere. The L_Q is the distance from the point B to the center of sphere.

Then we easily get the values of the Q and the length of the OB whose length is

represented by the L_Q if the OA length and the value of q are known. So we can know the distributing of the electric field intensity and the electrical potential in the sphere S₂. Here the electric potential of one point on the surface S₂ is not represented to the temperature at the point, because it is the isothermal sphere according to our conditions and the surface is not what we want to get. In fact we should get the electrical potential value in the interface S₁ between the air and the medium is the surface. See the picture 5a, the surface S₁ is the isothermal sphere that is labeled in the picture and we have got the interface S₂. We can get the electrical potential values of the interface S₂ if we get the length CD and then we shall

know the temperature value of the surface S_1 .

When we calculate the electric field intensity in the sphere S_2 we can ignore the layer from the S_1 to the S_2 because of the great difference between the ϵ_1 and the ϵ_2 . But we cannot ignore the layer if we want to get the electrical potential values of the interface S_2 .

□.CONCLUSIONS

At last we get two conclusions: the one is that the temperature distribution of the sphere thermal radiation is not affected if we consider the sphere as a plane whose temperature is not changed when we study the heat radiation from the medium surface to the detector. But we must remember the interface is not a plane, if we want to calculate the depth of the heat source. The picture 6 can validate the above result. The picture 6 is the temperature distributing image of an isothermal sphere, and the values are 197 and 198, whose error is enough little.

The values of the data in the picture 6 are 197, 198 and 199, and the black line is the manual boundary. Because there is the other object at the sphere top right corner that affect on the picture.

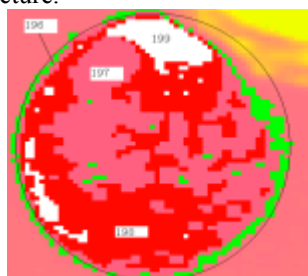


Figure 6.

The another conclusion is the interface isothermals are a group of homocentric circularities□and the picture 7 can test the result. When the temperature difference in the sphere is not great and the distance from the sphere to the detector is enough, we can try to use the enantiomorphous way to get the values in the interface between the air and the medium.

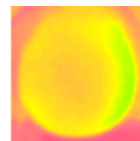
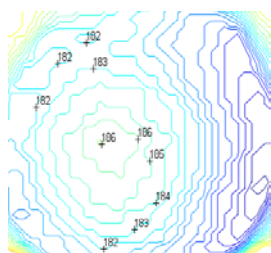


Figure 7

Although we get the conclusions, some problems still need to be solved. How should we get the parameters in the model, such as the length CD and the values of the ϵ_1 and the ϵ_2 ? Whether the model is different with the body? We shall continue to experiment in order to get the accurate results. And we have discover the intensity the detector receives is not related with the shape of the medium when we study the thermal radiation. We shall study the conclusion later.

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